Section One: Calculator-free **35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(4 marks)**

A function has a stationary point at (-2, 3) and is such that $f'(x) = 16 + kx^3$.

Determine the constant k and hence an equation for $f(x)$.

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Determine the constant k and hence an equation for $f(x)$.

$$16 + k(-2)^3 = 0$$
$$8k = 16 \Rightarrow k = 2$$

$$f'(x) = 16 + 2x^3$$
$$f(x) = 16x + \frac{x^4}{2} + c$$

$$3 = 16(-2) + \frac{(-2)^4}{2} + c$$
$$3 = -32 + 8 + c$$
$$c = 27$$

$$f(x) = 16x + \frac{x^4}{2} + 27$$

Question 6 **(7 marks)**

Let $f(x) = x^2 e^{2x-1}$.

(a) Determine the exact value of $f'(1)$. (3 marks)

(b) Use the increments formula $\delta y \approx \frac{dy}{dx} \times \delta x$ with $x = 1$ to estimate $f(1.02)$. (4 marks)

Question 6 **(7 marks)**

Let $f(x) = x^2 e^{2x-1}$.

(a) Determine the exact value of $f'(1)$. (3 marks)

$$f'(x) = (2x)\left(e^{2x-1}\right) + \left(x^2\right)\left(2e^{2x-1}\right)$$

$$f'(1) = (2)(e) + (1)(2e) = 4e$$

(b) Use the increments formula $\delta y \approx \frac{dy}{dx} \times \delta x$ with $x = 1$ to estimate $f(1.02)$. (4 marks)

When $x = 1,\ \frac{dy}{dx} = 4e$ and $\delta x = 0.02.$

$$\delta y \approx \frac{dy}{dx} \times \delta x$$
$$\delta y \approx 4e \times 0.02$$
$$\approx 0.08e$$

$$f(1) = e$$

$$f(1.02) \approx e + 0.08e$$
$$\approx 1.08e$$

2. (16 marks)

(a) Find the derivative of

(i) $f(x) = ln\left(\frac{x^2 - 3}{1 + x}\right)$ (2)

(ii) $g(x) = \frac{e^{sin(x)}}{cos(x)}$ (3)

(iii) $h(x) = e^x \times ln(x^2)$ (2)

(b) (i) Given $g(x) = \sqrt{sin(x)}$

show that $g'(x) = \dfrac{cos(x)}{2\sqrt{sin(x)}}$. (2)

(ii) Hence determine $\int -\dfrac{3\,cos(x)}{\sqrt{sin(x)}}\,dx$ (2)

(c) Find $2\int_0^4 \left(1-f\left(x\right)\right)dx$ given $\int_0^{10} f\left(x\right)dx=-6.4$ and $\int_4^{10} f\left(x\right)dx=2.3.$ (2)

(d) Given $r=\sqrt{t},\quad t=4x,\quad x=cos\left(\theta\right)$

find an expression for $\frac{dr}{d\theta}$ as a function of θ. (3)

Question 7 **(8 marks)**

A small storage tank, initially holding 20 L of water, is being filled so that the rate of change of volume of water in the tank t minutes after filling began is given by $\frac{dV}{dt} = \frac{10t}{t^2 + 1}$ for $t \geq 0$, where V is the volume of water in the tank, in litres.

(a) Use calculus to show that the tank is filling at the fastest rate after exactly 1 minute. (3 marks)

(b) Determine an expression for V in terms of t. (3 marks)

(c) The tank has a maximum capacity of 80 litres. Determine an exact expression for the time it will take to reach this capacity. (2 marks)

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(a) Use calculus to show that the tank is filling at the fastest rate after exactly 1 minute. (3 marks)

$$\frac{d^2V}{dt^2} = \frac{(10)(t^2+1)-(10t)(2t)}{\left(t^2+1\right)^2}$$

$$10t^2 + 10 - 20t^2 = 0$$
$$10 - 10t^2 = 0$$

$$t^2 = 1 \Rightarrow t = \cancel{-1},\ 1$$

(b) Determine an expression for V in terms of t. (3 marks)

$$V = \int \frac{10t}{t^2+1}\,dt$$
$$= 5\int \frac{2t}{t^2+1}\,dt$$

$$= 5\ln\left(t^2+1\right) + c$$

$$t = 0,\ V = 20 \Rightarrow c = 20$$
$$V = 5\ln\left(t^2+1\right) + 20$$

(c) The tank has a maximum capacity of 80 litres. Determine an exact expression for the time it will take to reach this capacity. (2 marks)

$$5\ln\left(t^2+1\right) + 20 = 80$$
$$\ln\left(t^2+1\right) = 12$$

$t = \sqrt{e^{12} - 1}$ minutes